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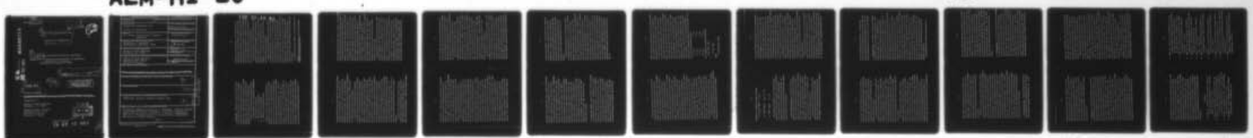

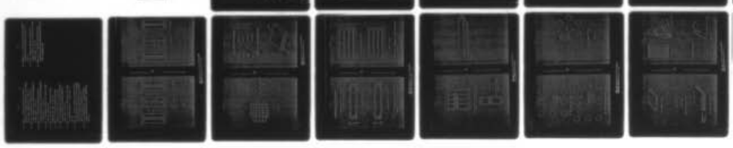

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STRUCTURAL INELASTICITY XX

Failure of Composites

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A review of composite materials is presented. Primary emphasis is on fiber-reinforced composites. Particular attention is paid to various failure mechanisms in tension, compression, and shear. Existing results are summarized and suggestions are given for future research.

1. Introduction. Composites in general may consist of two or more constituent materials. But most of the composites that are used in engineering applications are bi-phasic materials where a reinforcement phase is dispersed in a single continuous matrix phase. The physical properties of such a composite should be governed by the following factors:

a) geometry of the dispersoid such as shape and size, size distribution, volume concentration, concentration distribution, and orientation.

b) state of matter of the dispersoid.

c) composition of the dispersoid.

d) composition of the continuous phase.

However, the level to which each of these factors is accountable is dependent on the scale at which the material properties are evaluated. The knowledge of macroscopic properties that are obtained from the load-displacement data through conventional tests is sufficient for structural design purposes in case of metals and metallic alloys. But such limited knowledge may be inadequate for proper uses of composites. As a matter of fact, the macroscale behavior is a consequence of the molecular behavior which in turn is a reflection of the material behavior in the atomic scale. The scale can be further lowered to the sub-atomic range and so on. This is true for both metals (also metallic alloys) and composites. But composites introduce another intermediate scale in the form of fiber dimensions and interfaces. We

designate this as the microscale. The information of the composite behavior in the microscale is essential for true characterization of composites in the macroscale, with which structural engineers are concerned. Drucker [1][1] has pointed out that some important properties of composites in the macroscale may be completely masked, if the material behavior in the next lower scale is not properly explained.

Commonly used composites are broadly classified into three categories: dispersion strengthened, particle reinforced, and fiber reinforced. There is also another class of composite system known as eutectic or eutectoid such as lamellar composites. In the latter case, neither phase is a matrix, since both phases are quasi-continuum.

Dispersion strengthened composites are normally made by the powder metallurgy process. The disperse phase (dimensions of tiny particles vary from 0.01 - 0.1 μm) does not go into solution when the temperature is raised to the melting point of the matrix. The dispersoids are thus evenly distributed throughout the matrix medium. The concentration of the disperse phase is kept very low usually not more than 15% of the total composite volume. This type of composite draws its principal strength from the matrix material, but additional strength is derived since the dispersoid prevents motion of dislocations in the matrix.

The particle reinforced composite, on the other hand, is an

[1]Numbers in square brackets refer to the list of references collected at the end of the report.

intermediate stage between the dispersion strengthened composite and fiber reinforced composite. The concentration of the dispersoid is more than 25%, and the average diameter of the disperse particle is greater than 1 μm. Both the disperse and matrix phases contribute towards the strength of the composite. The strengthening mechanism is therefore a complex process involving interaction between the two phases.

A vast majority of the composites that are presently being used, however, belongs to the class of fiber reinforced composites. The fibers are usually the main load-bearing members. The matrix provides the necessary integrity and helps in transferring load from fiber to fiber through shear. There are certain additional complications in the analysis of these materials. One dimension of the fiber is much larger than the other two dimensions. This introduces anisotropy in fiber reinforced composites, whereas the other two composite systems (dispersion strengthened and particle reinforced) are essentially isotropic. Interestingly, the principal advantage of fiber reinforced composites is drawn from their anisotropic properties. The fiber orientations can be suitably tailored to specific advantage for optimum material utilization in regard to strength and stiffness of a structure. Unfortunately; inclusion of anisotropic behavior leads a lot of complications and mathematical involvement in the analysis procedures.

A composite may exhibit physical behavior altogether different from its constituent materials. We observe that a fiber-reinforced composite is anisotropic even though the fibers and matrix are isotropic. A composite containing randomly

oriented short fibers may again result in isotropy. Similarly homogeneous materials may combine to produce heterogeneous materials. Perfectly plastic constituents may produce appreciable work hardening. Perfectly elastic and perfectly brittle constituents may lead to plastic response. No plastic volume change in the microscale may give rise to a plastic volume change in the macroscale. Again components with very high creep rates may produce a material with very little creep. Drucker [1] cited several examples to substantiate the above statements in regard to composite material behavior. One interesting example shows how a composite with anisotropic elements may be isotropic in the next higher scale. Hot-rolled carbon structural steel consists of markedly anisotropic ferrite crystals and pearlite with its cementite platelets, but it exhibits macroscopic isotropy in the elastic range and is initially isotropic in the plastic range.

From now onwards, we limit our discussion mostly to fiber reinforced composites. The fibers and matrix may be either metallic or non-metallic or a combination of both. The non-metallic category may consist of either organic or ceramic materials. It is possible to use various combinations of fibers and matrices to produce a wide range of composites in the laboratory. Newer and more efficient composites are continuously being developed. However, in the practical realm the choice is still limited. The most commonly used fibers are Glass (E-glass and S-glass), Graphite, Boron, Kevlar-49 (organic fibers), FP (Polycrystalline Alumina),

and Beryllium. The matrices are thermoplastics (polyethelene, polystyrene, and nylon), thermosetting polymers (epoxy, polyester, phenolic, silicone, polyimide), graphite, and aluminum alloys. There are definite advantages and disadvantages of one material over the other, or those of one combination over another. The requirements for a specific application as well as fabrication and material cost form the basis for selection of a particular composite material system.

Different fibers may exhibit different strength and stiffness properties. A definite relationship between the strength (tensile) and stiffness (E, Young's modulus) of a material can be established based on the interatomic bonds. The tensile strength of a material should fall between 1/20 to 1/7 of its elastic modulus, yet most materials fail at lower stresses ranging from 0.001 E to 0.01 E [2]. Internal flaws such as microcracks and other material defects are the main causes leading to lower strengths. It is virtually impossible to produce a flawless material, but some of the defects in a material can be minimized in its fibrous form. Some microscopic flaws are inherent with the fiber and some are even added to the system during the fabrication process. This influences the physical properties of different fibers. Glass fibers usually possess low modulus and moderate strength properties. A wide range of material properties can be achieved in the graphite group of fibers, ranging from "high modulus-low strength" to "low modulus-high strength". Boron, Kevlar-49, and FP exhibit very high modulus and high strength. But, interestingly, the compressive strength of

Kevlar-49 is lower than its tensile strength, whereas the tensile strength of FP is lower than its compressive strength. Beryllium is considered to be the optimum material because of high specific strength and stiffness, although the cost and hazard involved in the manufacturing process may be prohibitive for most applications other than aerospace structures.

There are certain other factors that are of vital importance when one is concerned with failure of composites. Yielding, plastic flow, work hardening, etc. are common with metals and metallic alloys. Ceramics, in general, are brittle materials i.e., elastic up to fracture. They do not exhibit any sort of yielding prior to fracture, unless the temperature is raised very high, say, beyond one-half of their melting point. Thus it is observed [3] that the yield strength of glass fibers at 538°C is 0.85 GPa whereas the ultimate strength at the same temperature is 1.28 GPa. Highly cross-linked polymers are also brittle. Although they exhibit some yielding it is negligible compared to metals or to less brittle polymers such as thermoplasts.

The term 'failure' is very broad in its meaning. It may encompass a wide spectrum of physical behavior such as on-set of yielding, ultimate strength, crack nucleation and unstable crack growth, fracture, fatigue limit, flow due to creep phenomena, impact damage, and so on. Each failure pattern may have a distinctly different feature or diverse features coupled together. Diversity may increase with the composite material system used. The material composition, constituent material properties, anisotropy, interfacial bond, sequence of

laminations, fiber orientations, types of loadings, environmental conditions - all add to the complexities in the failure mechanism. Besides all these, the scale (micro or macro) of investigation is equally important, although it is emphasized earlier that one scale is complementary to the next higher or lower scale. This is therefore a very broad problem area. Some work [4,5] has been done on many of these aspects. Failures associated with both flawed and unflawed composites subjected to various loading conditions have been considered. The present review is however limited to failure of only unidirectional and laminated composites (unflawed) under simple loading conditions. By 'unflawed' composites we mean only those which do not have large size defects (cracks, voids, delaminations, etc.) so as to initiate failure due to stress concentrations. The responses due to other loading conditions such as fatigue, creep, and impact are also excluded from our preview.

2. <u>Failure under tension</u>. Tensile failure of fibrous composites will be treated in this section. The application of load is along the direction of the fiber. We may have four different possibilities: brittle fibers and brittle matrix, brittle fibers and ductile matrix, ductile fibers and brittle matrix, and ductile fibers and ductile matrix. In fiber reinforced composites loaded under tension, the nature and physical behavior of fibers however mostly govern the strength and failure modes of the overall composite. The fracture mode of failure is predominant in composites made with brittle

fibers and brittle matrix. Fracture defines the strength of a material with macroscopic flaws such as cracks, voids, debonding, etc. Fracture properties are investigated by stressing and analyzing precracked and/or notched specimens. The unflawed uniaxial composite however exhibits statistical variation in strength due to statistical distribution of microscopic flaws and imperfections inherent in brittle materials.

The statistical distribution of fiber strength leads to two importent considerations [6]: (i) the strength of a bundle of fibers is not necessarily equal to either the total strength of the individual fibers or to their mean value, and (ii) relatively low stress levels may also cause distributed failures. The degree of scatter in strength increases with the increase of brittleness of the composite. Thus, a composite with brittle fibers but ductile matrix will exhibit less scatter in tensile strength properties. The "rule of mixtures" cannot correctly predict the strength of a brittle composite. Hence the problem is to develop a model that relates the statistical strength of fibers to that of the composite. One such model [7] assumes that, as the load increases the fiber fractures at several points and the resulting accumulation of fiber fracture reduces the fiber-lengths to such extend that no more load transfer is possible because the matrix shear strength is exceeded. The final failure of the composite eventually occurs through the shear failure of the matrix (Fig. 1).

Another model assumes that the fibers are brittle but strong and stiff with respect to the matrix and their strength varies substantially from point to point along their length. When such a composite is stressed under a tensile load, the fiber breaks at one of the weakest flaws or imperfections. Subsequently there may develop several possibilities that may lead to final failure. In the vicinity of the fiber break, high interface shear stress will result causing interface failure along the length of the fiber. Secondly, the weak fracture toughness of the matrix may help in propagation of cracks both along and across the fiber leading to ultimate failure. Some investigators [8-9] have utilized this model for deriving the strength of a composite. This model predicts a lower stress level then the previous one [7]. If interface failure and matrix cracking are prevented, fiber fracture will occur in other weaker points, and this is what is usually desired (Fig. 2). A statistical tensile failure model (Fig. 3) of composites was proposed by Rosen [10]. Each fiber was assumed to be chain of n links. The length of each link is δ. Each layer is composed of bundle of such chains and the composite is a series of such bundles. Several other models [11-13] were also developed. We shall not discuss all these models here. The interested reader may refer to the original papers.

Composites made of ductile fibers and matrix, on the other hand, should fail in a ductile manner by plastic-flow instability. Relatively little is known about this aspect of composite

behavior compared to published work on fracture and statistical tensile strength of uniaxial composites. It is generally assumed that the "rule of mixtures" can predict the strength of a ductile composite fairly accurately. When a ductile composite (with perfectly plastic constituents) is loaded under tension, the material behavior consists of four different phases. Both fibers and matrix behave elastically during the initial stage of loading. The matrix goes to the plastic state upon further loading. As the loading increases, fibers also deform plastically. Lastly fibers fracture and cause final failure of composites accompained by some additional plastic flow. These stages are illustrated in Fig. 4. The assumptions of constant strain throughout the composite and perfectly plastic constituent material behavior simplify the analysis procedure. The "rule of mixtures" may therefore seem to give a reasonable estimate of the ultimate load. Thus for a fiber and the matrix we assume

$$\sigma_f = E_f\epsilon, \qquad \epsilon < \epsilon_{fy}$$

$$\sigma_f = \sigma_{fy}, \qquad \epsilon \geq \epsilon_{fy}$$

$$\sigma_m = E_m\epsilon, \qquad \epsilon < \epsilon_{my}$$

$$\sigma_m = \sigma_{my}, \qquad \epsilon \geq \epsilon_{my}$$

Now, assumption of constant strain gives

$$\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$$

or

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$

where

$$V_f = A_f/A_c \text{ and } V_m = A_m/A_c$$

Therefore, the composite strength is given by

$$\sigma_c = \epsilon_c[E_f V_f + E_m V_m], \qquad \epsilon_c < \epsilon_{my}$$

$$\sigma_c = \epsilon_c E_f V_f + \sigma_{my} V_m, \qquad \epsilon_{my} < \epsilon_c < \epsilon_{fy}$$

$$\sigma_c = \sigma_{fy} V_f + \sigma_{my} V_m, \qquad \epsilon_c > \epsilon_{fy}$$

These results are frequently used to estimate tensile strength of a unidirectional composite. But the real behavior is not as simple as this. A ductile composite should exhibit plastic flow and associated strain hardening. The experimental data and analytical work supporting this behavior are not available in the literature.

3. Failure due to compression. The loading direction is assumed to coincide with the fiber direction of the composite. There exist several possibilities through which a uniaxial composite may fail under axial compression: overall instability, local buckling, and stress exceeding the ultimate strength of the composite. The overall instability failure is commonly associated with the overall stiffness of the composite, and we rule out such possibility if we assume the composite to be stiff enough to overcome such failure.

The local buckling or micro-buckling is another type of compressive failure where the failure modes are highly localized within fibers alone but eventually may lead to final failure. This type of failure is common with a composite

having high stiffness fibers embedded in low stiffness matrix. The analogy is drawn to buckling of a column resting on an elastic foundation, Fig. 5a, or wrinkling of a soft-cored sandwich column under uniaxial compression, Fig. 5b. The thin faces of the sandwich column may be compared with the highly flexible narrow fibers. The matrix layer separating two adjacent fibers respresents the soft core of a sandwich column. It is therefore expected that a symmetric mode of buckling will occur when the separations between the fibers are large or in other words the composite has less fiber concentration. Similarly, an antisymmetric mode of buckling will be prevalent in a composite having closely spaced fibers. It will be of interest to study the effect of fiber volume concentration (also fiber-matrix properties) on these modes and on transition from one mode to the other. To the author's knowledge, the analogy between the wrinkling of the sandwich beam and micro-buckling modes (extensional and shear, Fib. 5c) of the composite has not been explored. Fairly extensive literature [11-14], however, exists on various other aspects of micro-buckling of composites.

Another form of micro-buckling i.e., kink-band formation of organic fibers (Kevlar-49) has also been reported [14]. The organic fibers are mostly fibrillar in structure, having very low shear stiffness and strength. This material property associated with the anisotropy of the fiber causes the formation of kink bands (Fig. 6a). An analogy with this type of failure can also be found with shear crimping of a sandwich column, Fig. 6b. The shear crimping is a phenomona associated with the

buckling of a sandwich column having very thin (therefore of very low transverse strength and stiffness) faces and soft core (very low shear strength and stiffness). In case of a sandwich column, however, the crimping form of buckling can be explained as one form of overall instability with very large or infinite wave numbers.

The micro-buckling behavior is mostly the guiding factor in defining the compressive strength of a composite. In reality, the limiting material strength of a composite under compression is hardly achieved. Composites fail at much lower stresses due to micro-instability. Further, the micro-buckling analysis procedures are applicable only in case of well-collimated and mostly large diameter fibers. The slightest eccentricity (which may be more severe for smaller diameter fibers) will considerably reduce the load-resisting capability of the composite.

Other factors such as residual stresses, interface debonding, and presence of micro-flaws will also influence the failure load. If the initial compressive yield strength of the fiber material is higher than that of the matrix material, the matrix may yield first at the interfaces. The process of yielding may be accelerated due to residual stresses that may exist at the interfaces. The fibers will then collapse due to loosening of the interface bonds, if the matrix yielding is further continued. Presence of micro-cracks may also initiate yielding by developing plastic zones at the tip of discontinuities where stress concentrations are high. An analysis taking into account all these essential ingredients will

be highly cumbersome and unmanageable. The models cited earlier [11-14] give satisfactory results for ideal cases only. The practical situation demands development of more realistic models and therefore there is much scope for further research.

4. Transverse failure. This refers to failure of composites loaded (both in tension or compression) perpendicular to the direction of fibers (Fig. 7). True characterization of transverse strength is important, because the transverse strength and axial strength (in the direction of fibers) of a composite vary greatly. The axial strength is primarily controlled by the concentration of fibers, whereas the transverse strength is dependent on the strength of the matrix. The composite strength is however higher than the matrix strength. Additional strengthening occurs, when the fibers restrict propagation of plastic flow (ductile matrix) and micro-cracks (brittle matrix). The shape, size, concentration, and distribution of the fibers and fiber-matrix interaction essentially control the additional strengthening mechanism. The behavior is more or less similar to that of particulate composites. Significant experimental work has been done to identify failure modes in brittle matrix particulate composites [15] and metal matrix particulate composites [16]. The works leading to theoretical interpretation of such experimental data are only a few. We discuss below some theoretical works which are essentially developed to determine transverse strength properties of filamentary composites.

The theoretical study is mostly concerned with the

interaction of fibers and matrix in the microscale. The usual practice is to assume some sort of regular arrays of circular fibers. The square array (Fig. 8a) and the hexagonal array (Fig. 8c) represent transverse isotropy, whereas the rectangular array (Fig. 8b) leads to transverse orthotropy. The repeatative blocks are then subjected to transverse load and analyzed.

Foye [17-18] utilized a finite element scheme to analyze the inelastic behavior of unidirectional composites and symmetric composite laminates. The loads were monotonically increased, but the initial load ratios were maintained throughout the analysis. No attempt however was made to study the behavior during unloading. Miller and Adams [19] presented a pseudo-three dimensional analysis (assuming generalized plane strain conditions) to determine the elastic-plastic response of the composite under transverse loadings. The essential features of a plastic analysis have not been accounted for and therefore the analysis presented in their report is merely a nonlinear elastic one. The true plastic behavior must include the response due to loading and unloading, and subsequent modification of yield surfaces. Most matrix materials are expected to produce much more complex unloading curves than those of metals and metallic alloys. Such possibilities have not been explored either experimentally or analytically.

5. Failure due to shear. This constitutes failures due to shear in the plane of the composite ply (Fig. 9a) and shear acting normal to the plane of the ply (Fig. 9b). In the

first case, very little resistance is provided by the reinforcements and the composite shear strength is determined from the shear strength of the matrix. In the second case, however, the reinforcements definitely contribute towards the composite shear strength. But, if there exists any weaker plane such as an interlaminar layer (Fig. 9c), the transverse shear strength will closely approximate to the interlaminar shear strength of the laminated media. The interlaminar shear failure originates at the plane of maximum transverse shear stress. An experimental method such as the rail shear test can be utilized to determine the inplane shear strength of the composite. Short beam shear tests are usually employed to evaluate interlaminar shear strength.

6. Laminate failure. We have so far discussed failure associated with one-dimensional stress system. More complex failure mechanisms may develop if the loading results in a combined stress field. Such a situation may arise in case of an off-axis (the fiber and loading directions do not coincide) composite specimen, even if a uniaxial load is applied. The situation is more complex, and the stress system is a three-dimensional one if a uniaxial load is applied to a laminate consisting of laminas having different fiber orientations. To determine failure load for such a case, the common practice is to define a failure criteria (strength criteria) for each lamina and then predict the progressive layer to layer failure. The logical first step that was followed, was therefore to extend Hill's

anisotropic failure criteria [19] to composites. This was carried out by Azzi and Tsai [20]. Hill's criteria was further generalized by Hoffman [21] to account for unequal tensile and compressive strength of the composite. References 22-27 deal with various strength theories of composites. The most important drawback with all these failure theories is that it is tacitly assumed that yielding and final failure of a lamina occur at the same moment. This is however not true in the practical case.

7. Conclusions. In the previous sections, we have touched certain features associated with failure of composites. We have however avoided discussing various other aspects such as failure due to fracture, crack growth, fatigue, creep, and so on. Crazing is another interesting phenomena that may cause premature failure of polymeric composites. Crazes are formed when a polymer is subjected to stress. A tensile stress will cause crazes to be developed along perpendiculars to the loading direction. Crazes are not cracks, although they are of similar appearance. They consist of thin bonds (about 1 μm thick) of porous polymers. The polymer content is normally about 50 per cent in the crazed area. The material of the crazed area may undergo a deformation of 100% or even more during the process of crazing. Interestingly, the crazed portion of the material is capable of resisting load and may be as strong as the uncrazed material. But it is expected that the crazed material will exhibit more pronounced inelastic material behavior, as the crazed part is softer than the

uncrazed one. The behavior is most likely to be influenced by temperature, especially near the range of the glass transition temperature. Due to its porous nature the crazed polymer is also susceptible to moisture environments. It is therefore suggested that crazing in polymers and failure of polymeric composites due to crazing should be investigated properly.

Plastic behavior and associated work hardening is another important area which should receive more attention. A very few analyses have appeared in the literature dealing with this subject matter. Then again they do not deal with the true plastic behavior. In all these analyses, the material behavior is assumed to be linear or piecewise-linear up to the ultimate failure. The effect of loading, unloading, and reloading has not been considered. A wide class of metal composites will definitely show strain hardening characteristics. The particulate composite, where the matrix may be either metal or polymer, is likely to exhibit plastic behavior. Plastic flow may also occur in filamentary composites under action of transverse tension, compression, or shear. Moisture in polymeric composites and high temperatures may aggravate the situation. Fig. 10 is self-explanatory in showing how failure of even brittle fibers (Fig. 10a) and opening and closing of cracks in brittle materials (Fig. 10b) may develop plastic behavior.

Strain hardening is a complex phenemona. It is not fully understood for metals and metallic alloys even after several years of research and experience. But the importance of its study remains. Some beginning should also be made to understand work hardening phenomena in composites. The development of a

most general theory for an initially anisotropic material, such as a composite, will be a difficult task in the beginning stage. But to start with we may concentrate on a simpler theoretical model as suggested below. It may be assumed that the matrix material is initially isotropic, that the anisotropy introduced by hardening in the matrix is known, that fibers harden with a known Baushinger effect between tension and compression, and that all these components are piecewise-linear [28]. The model may then permit inclusion of various degrees of generality in the matrix behavior. Any theoretical model so developed should have necessary experimental support in view of the expected wide scatter in the experimental data of composites.

REFERENCES

1. Drucker, D.C., "Yielding, Flow and Failure" in Inealstic Behavior of Composite Materials (ed. C.T. Herakovich), AMD-Vol. 13, ASME Publi., 1975, 1.
2. Kelly, A., Strong Solids, Clarendon Press, Oxford, 1966.
3. Lowrie, R.E., "Glass Fibers for High-strength Composites", in Modern Composite Materials (eds. L.J. Broutman and R.H. Krock), Addison-Wesley Publ. Co., London, 1967, 281.
4. Broutman, L.J. and Krock, R.H. (eds.), Composite Materials, Vols. 1-8, Academic Press, N.Y. , 1974.
5. Herakovich, C.T. (ed.), Inelastic Behavior of Composite Materials, AMD-Vol. 13, ASME Publ., 1975.

6. Rosen, B.W., Kulkarni, S.V., and McLaughlin, Jr., P.V., "Failure and Fatigue Mechanisms in Composite Materials", in Inelastic Behavior of Composite Materials (ed. C.T. Herakovich), AMD-Vol. 13, ASME Publ., 1975, 17.
7. Parrat, N.J., "Defects in Glass Fibers and Their Effect on the Strength of Plastic Mouldings", Rubber and Plastic Age, 1, 1960, 263.
8. Rosen, B.W., Dow, N.F., and Hashin, Z., Mechanical Properties of Fibrous Composites, NASA CR-31, April 1964.
9. Hedgepeth, J.M., Stress Concentrations in Filamentary Structures, NASA TN D-882, May 1961.
10. Rosen, B.W., "Tensile Failure of Fibrous Compositrs", AIAA J., 2, 1964, 1985.
11. Rosen, B.W., "Mechanics of Composite Strengthening" in Fiber Composite Materials, American Society for Metals, 1965, 37.
12. Schuerch, H., "Prediction of Compressive Strength in Uniaxial Boron Fiber-Metal Matrix Composite Materials", AIAA J, 4, 1966, 102.
13. Greszczuk, L.B., Failure Mechanics of Composites Subjected to Compressive Loading, AFML-TR-72-107, 1972.
14. Kulkarni, S.V., Rice, J.S., and Rosen, B.W., An Investigation of the Compressive Strength of PRD-49-III/Epoxy Composites, NASA CR-112334, 1973.
15. Lange, F.F., "Fracture of Brittle Matrix, Particulate Composites", in Composite Materials (eds. L.J. Broutman and R.H. Krock), 5, 1974, 1.

16. Gurland, J., "Fracture of Metal-Matrix Particulate Composites", in Composite Materials (eds. L.J. Broutman and R.H. Krock), Academic Press, 5, 1974, 45.

17. Foye, R.L., "Theoretical Post-Yielding Behavior of Composite Laminates, Part I. Inelastic Micromechanics", J. Compos. Mater., 7, 1973, 178.

18. Foye, R.L. "Theoretical Post-Yielding Behavior of Composite Laminates, Part II. Inelastic Micromechanics", J. Compos. Mater., 7, 1973, 310.

19. Hill, R., The Mathematical Theory of Plasticity, Oxford Univ. Press, London, 1950.

20. Azzi, V.D. and Tsai, S.W., "Anisotropic Strength of Composites, Expt. Mech., 5, 1965, 283.

21. Hoffman, O., "The Brittle Strength of Orthotropic Materials", J. Compos. Mater., 1, 1967, 200.

22. Petit, P.H. and Waddoups, M.E., "A Method of Predicting the Nonlinear Behavior of Laminated Composites, J. Compos. Mater., 3, 1969, 2.

23. Tsai, S.W. and Wu, E.M., "A General Theory of Strength for Anisotropic Materials", J. Compos, Mater., 5, 1971, 58.

24. Sandhu, R.S., "Nonlinear Response of Unidirectional and Angle-ply Laminates", AIAA Paper No. 74-380, Presented at 15th AIAA-ASME Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974.

25. Wu, E.M. and Scheublein, J.K., "Laminate Strength-A Direct Characterization Procedure", ASTM STP 546, 1974, 188.

26. Sendeckyj, G.P., "A Brief Survey of Emperical Multiaxial Strength Criteria for Composites", ASTM STP 497, 1972, 41.

27. Wu, E.M., "Strength and Fracture of Composites" in Composite Materials (eds. L.J. Broutman and R.H. Krock) Academic Press, 5, 1974, 191.

28. Hodge, P.G., Jr., A Review of Some Piecewise Linear Theories of Plastic Strain hardening, Report AEM-H1-11, Univ. of Minnesota, 1974.

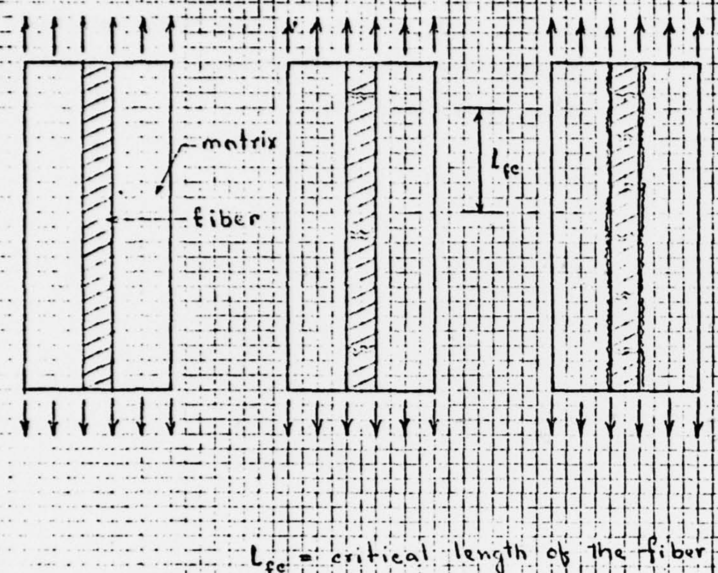

matrix
fiber
l_{fc}
l_{fc} = critical length of the fiber

Fig. 1

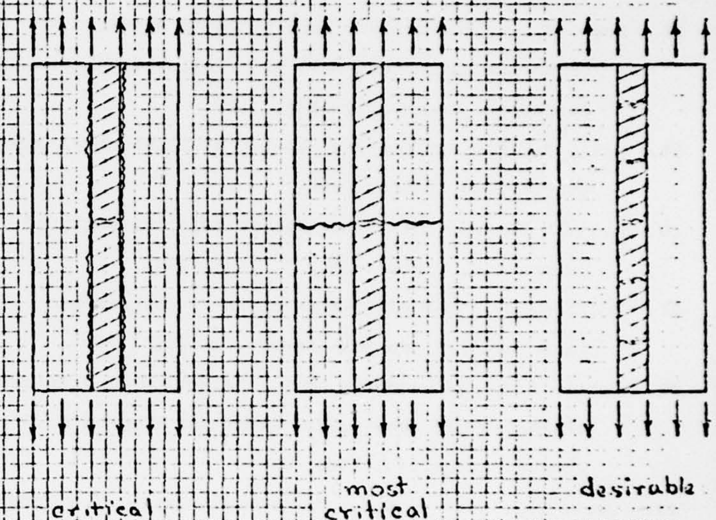

critical
most critical
desirable

Fig. 2

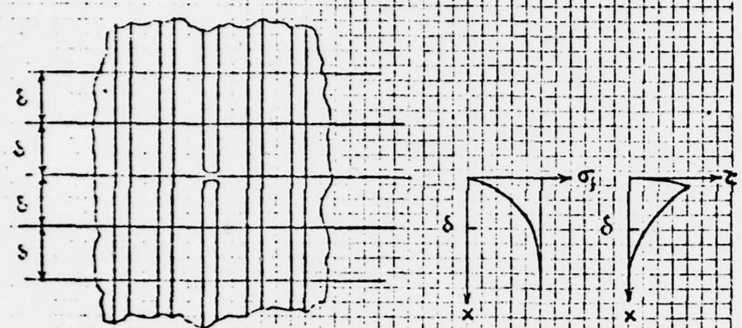

ε
s
ε
s
σ_f
δ
x
τ

Fig. 3

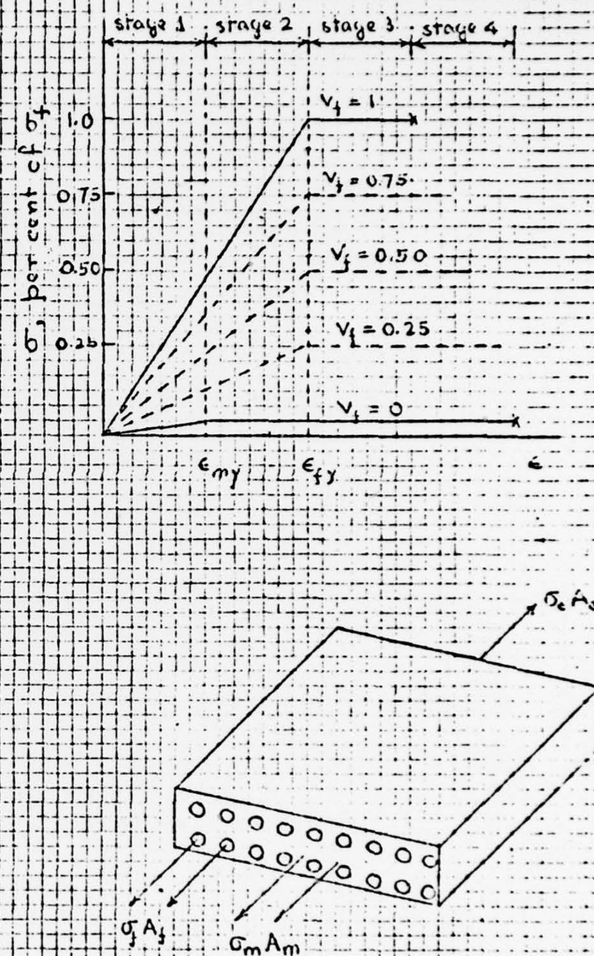
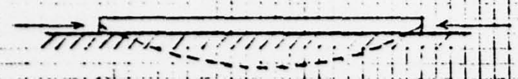
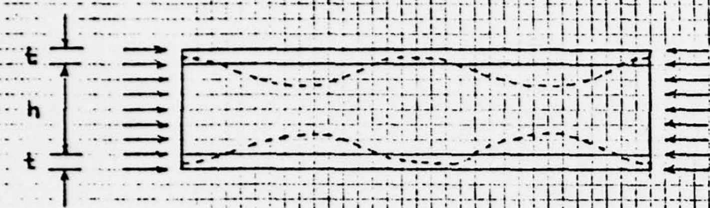

stage 1
stage 2
stage 3
stage 4
σ per cent of σ_f
1.0
0.75
0.50
0.25
$V_f = 1$
$V_f = 0.75$
$V_f = 0.50$
$V_f = 0.25$
$V_f = 0$
ϵ_{my}
ϵ_{fy}
ϵ
$\sigma_c A_c$
$\sigma_f A_f$
$\sigma_m A_m$

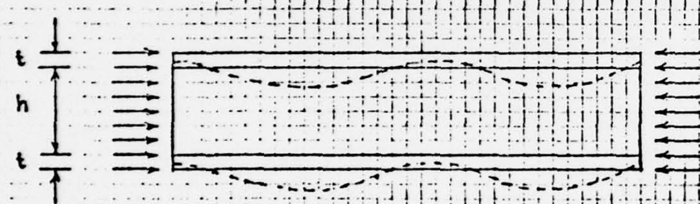
Fig 4



(a) Buckling of a column resting on an elastic foundation



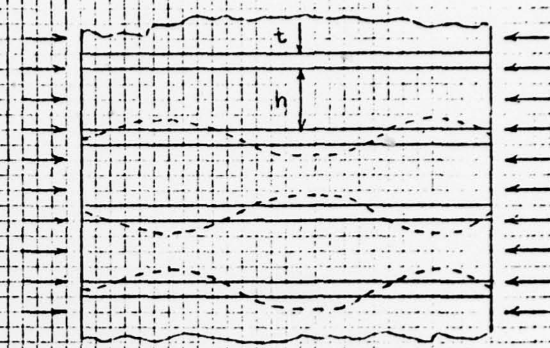
Symmetric mode



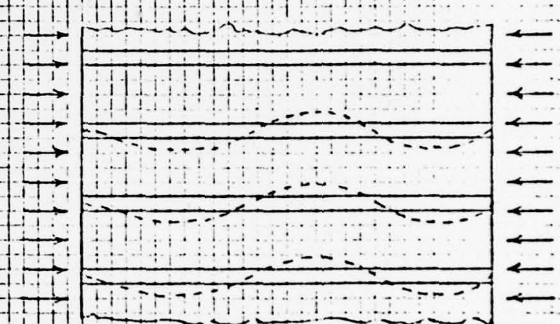
Antisymmetric mode

(b) Wrinkling of soft-cored sandwich column (two distinct modes)

FIG. 5



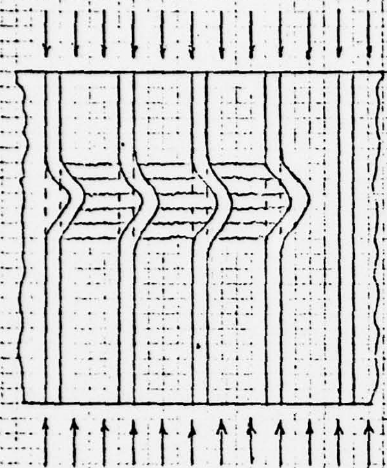
Extensional mode



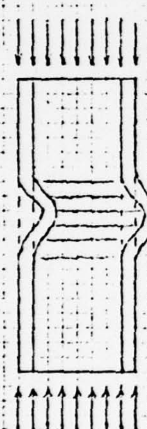
Shear mode

(c) Micro buckling of fiber-reinforced composites.

FIG. 5



(a) Kink-band formation in fiber-reinforced composites



(b) Shear crimping in sandwich column.

FIG. 6

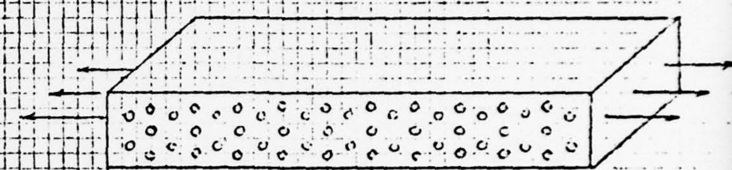
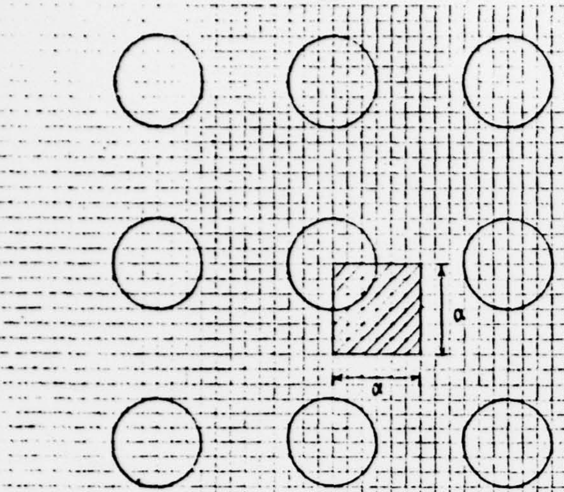
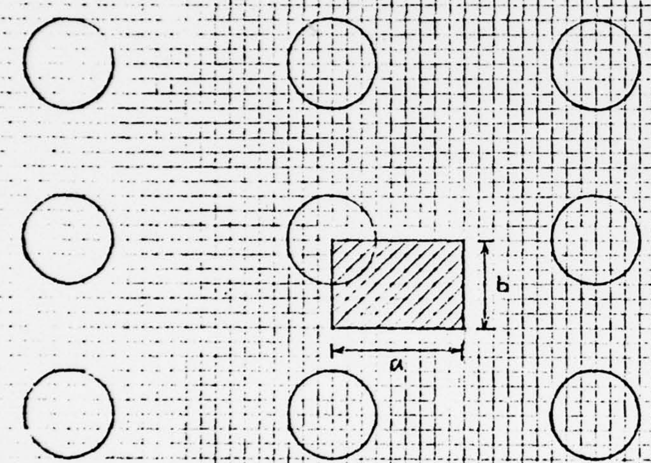


FIG. 7

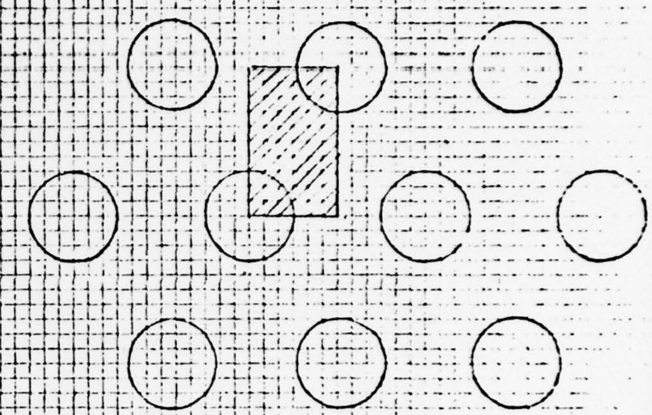


(a) Square array



(b) Rectangular array

Fig 8



(c) Hexagonal array

Fig 8

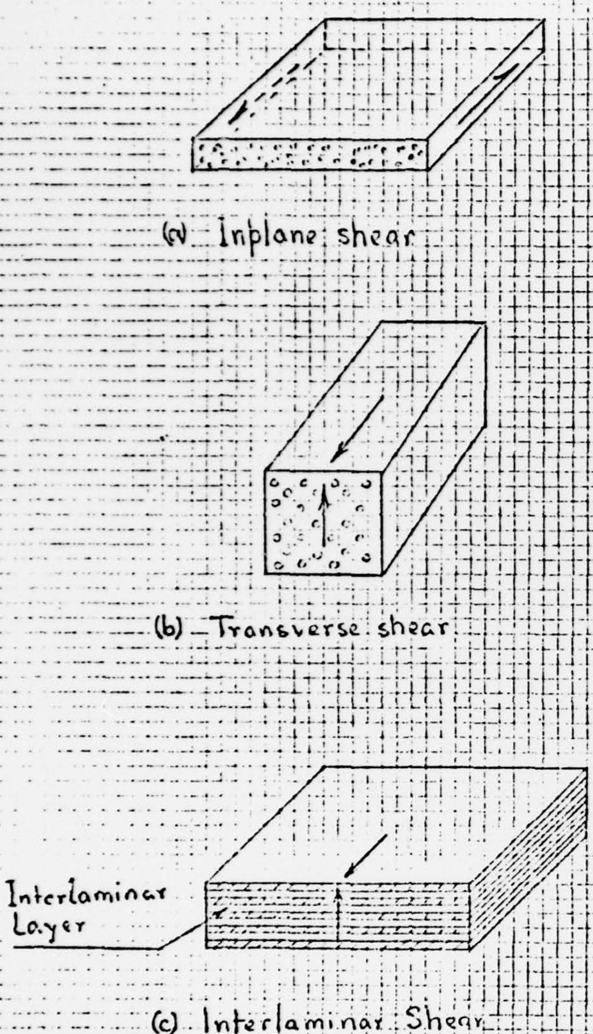


Fig. 9

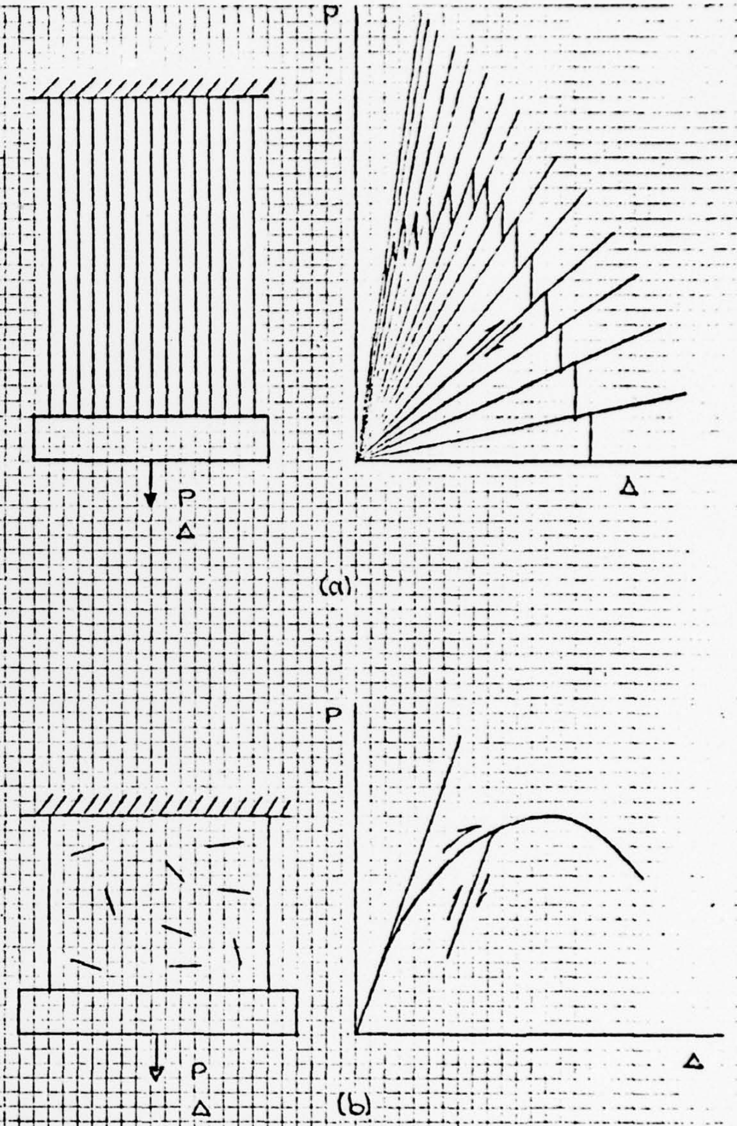


Fig. 10 [Ref. 1]